

YEAR 8 — PROPORTIONAL REASONING...

Block 1: Ratio and Scale

What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved

Keywords

Ratio: a statement of how two or more numbers compare

Equal Parts: all parts in the same proportion, or a whole shared equally

Order: to place a number in a determined sequence

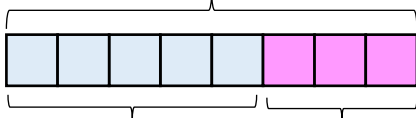
Part: a section of a whole

Factors: integers that multiply together to get the original value

Representing a ratio

"For every 5 boys there are 3 girls"

This is the "whole" — boys and girls together



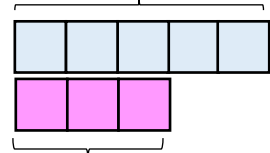
This represents the 5 boys

This represents the 3 girls

5:3

This represents the 5 boys

Double Number Line



This is the "whole" — boys and girls together

This represents the 3 girls

Order is Important

"For every dog there are 2 cats"



Dogs: Cats
1:2

The ratio has to be written in the same order as the information is given

e.g. 2:1 would represent 2 dogs for every 1 cat ✗

Simplifying a ratio

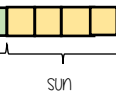
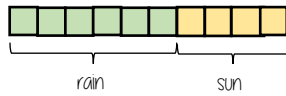
Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

+ by 2 ↓

3:2



Find the biggest common factor that goes into all parts of the ratio

For 6 and 4 the biggest factor (number that multiplies into them is 2)

"For every 3 days of rain there are 2 days of sun" — when this happens twice the ratio becomes 6:4.

Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit. Therefore Divide by 4

4 : 20
1 : 5

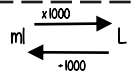
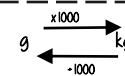
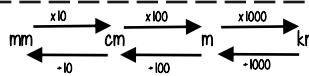
This side has to be divided by 4 too — to keep in proportion

**The n part does not have to be an integer for this type of question

Units are important:

When using a ratio — all parts should be in the same units

Useful Conversions



Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Lucy

£350 ÷ 7 = £50

□ = one part = £50

Find the value of one part

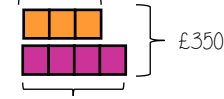
Whole: £350
7 parts to share between (3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 x £50 = £150

(x 50) 3 : 4 (x 50)
→ £150 : £200



Lucy = 4 x £50 = £200

Finding a value given 1:n (or n:1)

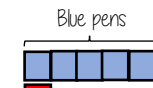
Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

Model the Question

Blue: Red

5 : 1

□ = one part = 10 pens



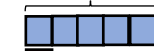
Red pens

One unit = 10 pens

Put back into the question

Blue pens = 5 x 10 = 50 pens

Blue: Red
(x 10) 5 : 1 (x 10)
→ 50 : 10



Red pens = 1 x 10 = 10 pens

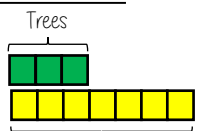
There are 50 Blue Pens

Ratio as a fraction



Trees: Flowers

3 : 7



Ratio

There are 3 parts for trees

Fraction of trees

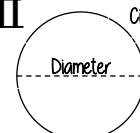
Number of parts in group
Total number of parts

3
10

Fraction

Trees parts 3 + Flower parts 7 = 10

π



Circumference

Diameter

The ratio of a circles circumference to its diameter

YEAR 8 — PROPORTIONAL REASONING...

Block 2: Multiplicative Change

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions
- Understand and use scale factors for length

Keywords

Proportion: a statement that links two ratios

Variable: a part that the value can be changed

Axes: horizontal and vertical lines that a graph is plotted around

Approximation: an estimate for a value

Scale Factor: the multiple that increases/ decreases a shape in size

Direct Proportion

As one variable changes the other changes at the same rate.



4 cans of pop = £2.40

4 cans of pop = £2.40
 $\times 0.5$
 2 cans of pop = £1.20

This is a multiplicative change

4 cans of pop = £2.40

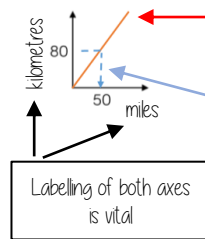
12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first
 e.g. 1 can of pop = £0.60

This multiplier is the same in the same way that this would be for ratio

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare — then find the associated point by using your graph. Using a ruler helps for accuracy. Showing your conversion lines help as a "check" for solutions

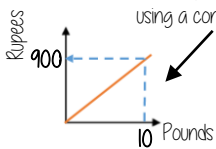
Conversion between currencies



£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees



Currency can be converted using a conversion graph

Convert 630 Rupees into Pounds

£1 = 90 Rupees
 $\times 7$
 £7 = 630 Rupees

Ratio between similar shapes



Angles in similar shapes do not change
 e.g. if a triangle gets bigger the angles can not go above 180°

The two rectangles are similar.



Corresponding sides

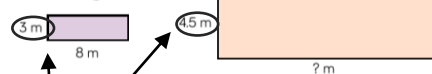
3m : 4.5m
 1m : 1.5m

8m : 12m
 1m : 1.5m

Note: Simplify to the same ratio

Understand Scale Factor

The two rectangles are similar.



$$3 \times 1.5 = 4.5$$

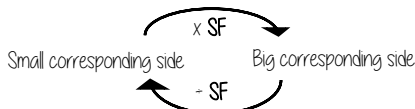
This is a multiplicative change.

Use corresponding sides to calculate a scale factor

Missing length
 $8 \times 1.5 = 12m$

Scale factor can also be calculated by:

Bigger corresponding side
Smaller corresponding side



Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life
 1cm : 30cm
 $\times 10$
 10cm : 300cm

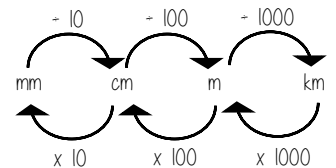


The car in real life is 210cm

Image : Real life
 1cm : 30cm
 $\times 7$
 7cm : 210cm



Interpret maps with scale factors



1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

1 cm : 25000cm

$$250 \times 100 = 25000$$

For every 1cm on my map is 25000cm in real life



YEAR 8 — PROPORTIONAL REASONING...

Block 3: Multiplying and Dividing Fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Carry out any multiplication or division using fractions and integers.
- Solutions can be modelled, described and reasoned.

Keywords

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken.

Denominator: the number below the line on a fraction. The number represent the total number of parts.

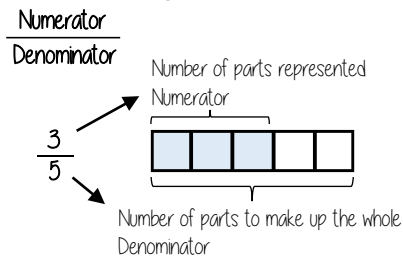
Unit Fraction: a fraction where the numerator is one and denominator a positive integer.

Non-unit Fraction: a fraction where the numerator is larger than one.

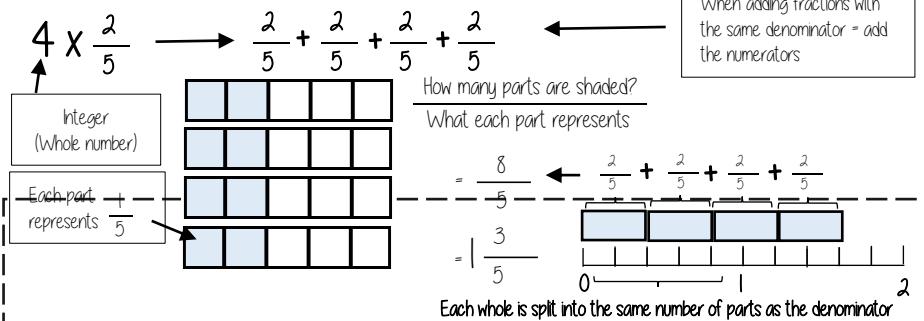
Reciprocal: a pair of numbers that multiply together to give 1.



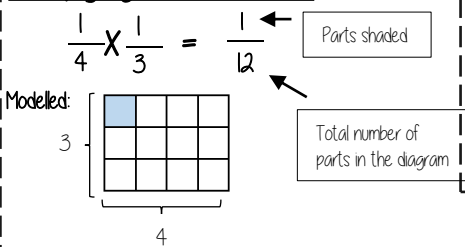
Representing a fraction



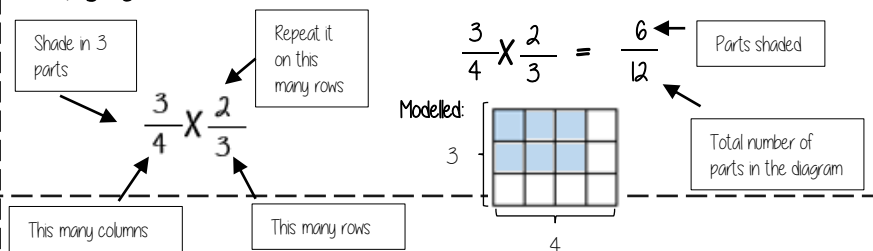
Repeated addition = multiplication by an integer



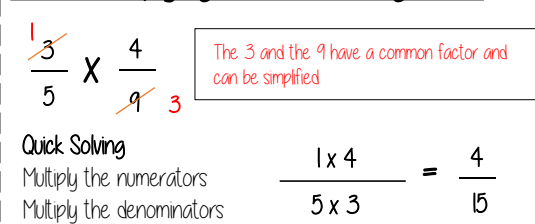
Multiplying unit fractions



Multiplying non-unit fractions

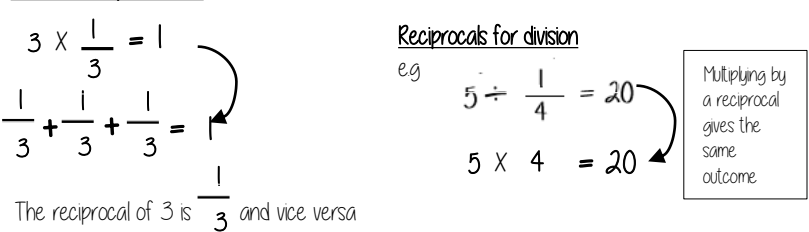


Quick Multiplying and Cancelling down

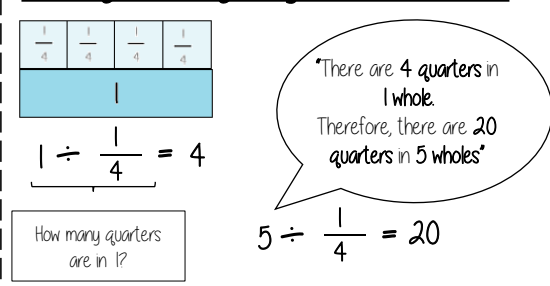


The reciprocal

When you multiply a number by its reciprocal the answer is always 1

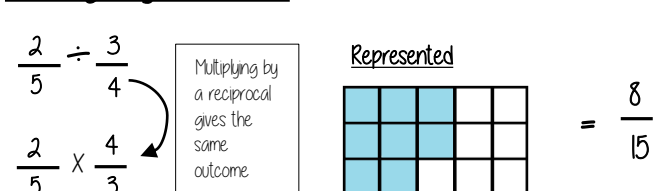


Dividing an integer by an unit fraction



Dividing any fractions

Remember to use reciprocals



YEAR 8 — REPRESENTATIONS...

Block 4: Representing Data

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and interpret scatter graphs
- Describe correlation and relationships
- Identify different types of non-linear relationships
- Design and complete an ungrouped frequency table
- Read and interpret grouped tables (discrete and continuous data)
- Represent data in two way tables

Keywords

Variable: a quantity that may change within the context of the problem

Linear: increases or decreases at the same rate

Relationship: the link between two variables (items) E.g Between sunny days and ice cream sales

Correlation: the mathematical definition for the type of relationship.

Outlier: a point that lies outside the trend of graph

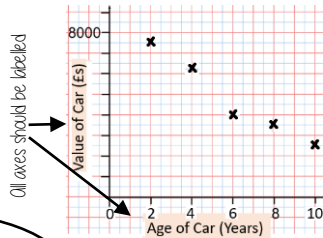
Quantitative: numerical data

Qualitative: descriptive information, colours, genders, names, emotions etc.

Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (Es)	7500	6250	4000	3500	2500

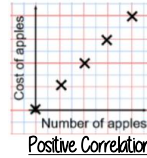
- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship



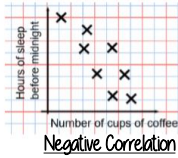
"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

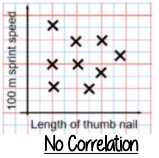
Linear Correlation



As one variable increases so does the other variable



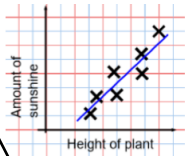
As one variable increases the other variable decreases



There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph



It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

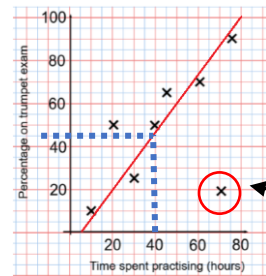
Things to know:

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point

e.g 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data

This is not always useful — in this example you cannot score more than 100%. So revising for longer can not be estimated

This point is an "outlier" it is an outlier because it doesn't fit this model and stands apart from the data

Ungrouped Data

The number of times an event happened

The table shows the number of siblings students have. The answers were
3, 1, 2, 2, 0, 3, 4, 1, 1, 2, 0, 2

2 people had 0 siblings. This means there are 0 siblings to be counted here

Number of siblings	Frequency
0	2
1	3
2	4
3	2
4	1

0 → 0
3 → 3
2 + 2 + 2 + 2 OR 2 x 4 = 8
3 + 3 OR 3 x 2 = 6
4 → 4

Best represented by discrete data (Not always a number)

2 people have 3 siblings so there are 6 siblings in total

OVERALL there are
0 + 3 + 8 + 6 + 4
Siblings = 21 siblings

Grouped Data

If we have a large spread of data it is better to group it. This is so it is easier to look for a trend. Form groups of equal size to make comparison more valid and spread the groups out from the smallest to the largest value.

Cost of TV (£)	Tally	Frequency
101 - 150	THH	7
151 - 200	THH THH	11
201 - 250	THH	5
251 - 300		3

We do not know the exact value of each item in a group — so an estimate would be used to calculate the overall total (Midpoint)

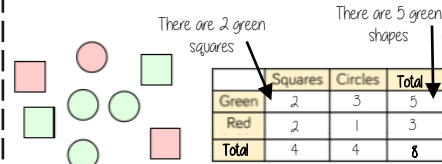
x	Frequency
Weight(g)	
40 < x ≤ 50	1
50 < x ≤ 60	3
60 < x ≤ 70	5

To make sure all values are included inequalities represent the subgroups

e.g this group includes every weight bigger than 60kg, up to and including 70kg

Representing data in two-way tables

Two-way tables represent discrete information in a visual way that allows you to make conclusions, find probability or find totals of sub groups



Using your two-way table

To find a fraction
e.g What fraction of the items are red? 3 red items
but 8 items in total = $\frac{3}{8}$

Interleaving: Use your fraction, decimal percentage, equivalence knowledge

YEAR 8 — REPRESENTATIONS...

Block 5: Tables and Probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct a sample space diagram
- Systematically list outcomes
- Find the probability from two-way tables
- Find the probability from Venn diagrams

Keywords

Outcomes: the result of an event that depends on probability

Probability: the chance that something will happen

Set: a collection of objects

Event: the outcome of a probability — a set of possible outcomes

Biased: a built in error that makes all values wrong by a certain amount

Construct sample space diagrams



Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from tossing a coin

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

This is the set notation to list the outcomes $S =$

$$S = \{1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T\}$$

In between the $\{ \}$ are a_i the possible outcomes

Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

What is the probability that an outcome has an even number and a tails?

This is the set notation that represents the question P

$$P(\text{Even number and Tails}) = \frac{3}{12}$$

In between the $()$ is the event asked for

There are three even numbers with tails

Numerator: the event

Denominator: the total number of outcomes

There are twelve possible outcomes

Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

$$P(\text{Girl walk to school}) = \frac{21}{100}$$

The event

The total in the set

The total number of items

Product Rule

The number of items in event a

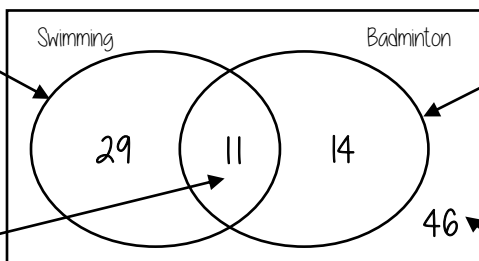
x

The number of items in event b

Probability from Venn diagrams

100 students were questioned if they played badminton or went to swimming club
40 went swimming, 25 went to badminton and 11 went to both

This whole curve includes everyone that went swimming
Because 11 did both we calculate just swimming by $40 - 11$



This whole curve includes everyone that went to badminton
Because 11 did both we calculate just badminton by $25 - 11$

$$P(\text{Just swimming}) = \frac{29}{100}$$

The intersection represents both
Swimming AND badminton

The number outside represents those that did neither badminton or swimming

$$100 - 29 - 11 - 14$$

YEAR 8 — ALGEBRAIC TECHNIQUES...

Block 6: Indices

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Keywords

Base: The number that gets multiplied by a power

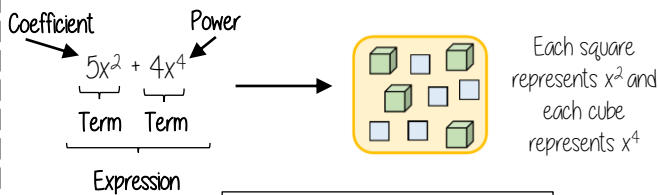
Power: The exponent — or the number that tells you how many times to use the number in multiplication

Coefficient: The number used to multiply a variable

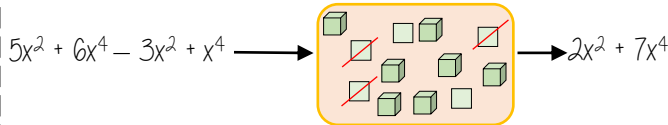
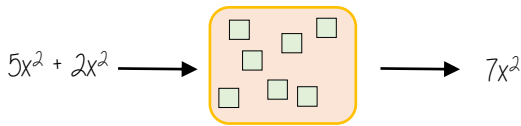
Simplify: To reduce a power to its lowest term

Product: The answer when numbers are multiplied

Addition/ Subtraction with indices



Only similar terms can be simplified
If they have different powers, they are unlike terms



Multiply expressions with indices

$$\begin{aligned} 4b \times 3a & \\ \equiv 4 \times b \times 3 \times a & \\ \equiv 4 \times 3 \times b \times a & \\ \equiv 12ab & \end{aligned}$$

$$\begin{aligned} 5t \times 9t & \\ \equiv 5 \times t \times 9 \times t & \\ \equiv 5 \times 9 \times t \times t & \\ \equiv 45t^2 & \end{aligned}$$

$$\begin{aligned} 2b^4 \times 3b^2 & \\ \equiv 2 \times b \times b \times b \times b \times 3 \times b \times b & \\ \equiv 2 \times 3 \times b \times b \times b \times b \times b \times b & \\ \equiv 6b^6 & \end{aligned}$$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$$3^5 \times 3^2 \longrightarrow 3^7$$

$$= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$$

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$$3^5 \div 3^2 \longrightarrow 3^3$$

$$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3} \longrightarrow \frac{3^3}{3^0} \longrightarrow \frac{3^3}{1}$$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

Divide expressions with indices

$$\frac{24}{36} \longrightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \longrightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \longrightarrow \frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b} \longrightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\left. \frac{23a^7y^2}{5db^6} \right\} \text{ This expression cannot be divided (cancelled down) because there are no common factors or similar terms}$$

YEAR 8 — REPRESENTATIONS...

Block 7: Co-ordinates

What do I need to be able to do?

By the end of this unit you should be able to:

- Plot and read co-ordinates in all four quadrants

Keywords

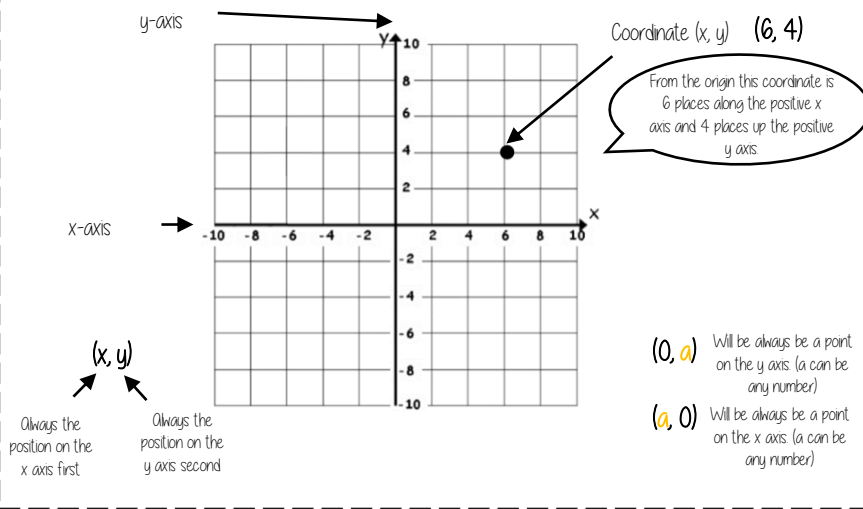
Quadrant: four quarters of the coordinate plane.

Horizontal: a straight line from left to right (parallel to the x axis)

Vertical: a straight line from top to bottom (parallel to the y axis)

Origin: (0,0) on a graph. The point the two axes cross

Coordinates in four quadrants



YEAR 8 — ALGEBRAIC TECHNIQUES...

Block 8: Brackets, Equations & Inequalities

What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

Keywords

- Simplify:** grouping and combining similar terms
Substitute: replace a variable with a numerical value
Equivalent: something of equal value
Coefficient: a number used to multiply a variable
Product: multiply terms

Form expressions

For unknown variables, a letter is normally used in its place

More than — ADD


Less than/ difference — SUBTRACT

e.g. 4 more than t \longrightarrow $t + 4$
 8 less than k \longrightarrow $k - 8$

Only similar terms can be grouped together

e.g. Find the perimeter of this shape

(Perimeter = length around outside of shape)

t  $t + 2t + 1 + t + 2t + 1 \longrightarrow 6t + 2$

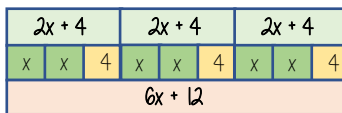
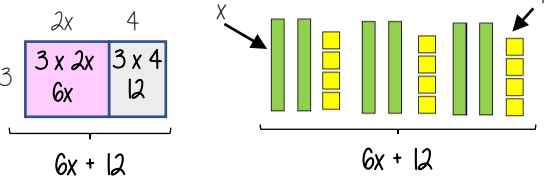
Directed numbers

$++ \longrightarrow +$
 $-- \longrightarrow +$
 $+ - \longrightarrow -$
 $- + \longrightarrow -$

e.g. $a = -5$ and $b = 2$
 $a^2 = a \times a = -5 \times -5 = 25$
 $b + a = 2 + -5 = -3$

Multiply single brackets

$3(2x + 4)$



Different representations of $3(2x+4) = 6x + 12$

Factorise into a single bracket

$8x + 4$



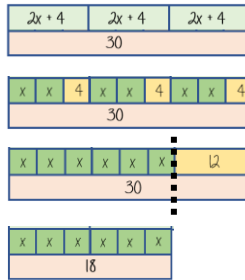
The two values multiply together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
 $8x + 4 \equiv 2(4x + 2)$
 This is factorised but the HCF has not been used

Solve equations with brackets

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

-12

$6x = 18$

-6

Substitute to check your answer.
 This could be negative or a fraction or decimal

$\frac{x}{3} = 3$

Simple Inequalities

- $<$ less than \leq Less than or equal to
 $>$ More than \geq More than or equal to

$x < 10$

Say this out loud
 "x is a value less than 10"

$10 > x$

Say this out loud
 "10 is more than the value"

Note:
 $x < 10$ and $10 > x$
 represent the same values

$x + 2 \leq 20$

"my value + 2 is less than or equal to 20"

$x \leq 18$

The biggest the value can be is 18

Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

Form

$x \longrightarrow x3 \longrightarrow +2 \longrightarrow 11$

$3x + 2 > 11$

Solve

$x \longleftarrow -3 \longleftarrow -2 \longleftarrow 11$

$x > 3$

Check

This would suggest any value bigger than 3 satisfies the statement

$3 \times 3 + 2 = 11 \checkmark$

$10 \times 3 + 2 = 32 \checkmark$

Algebraic constructs

Expression

A sentence with a minimum of two numbers and one maths operation

Equation

A statement that two things are equal

Term

A single number or variable

Identity

An equation where both sides have variables that cause the same answer includes \equiv

Formula

A rule written with all mathematical symbols e.g. area of a rectangle $A = b \times h$

YEAR 8 — ALGEBRAIC TECHNIQUES...

Block 9: Sequences

What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the n th term
- Recognise geometric sequences and other sequences that arise

Keywords

Sequence: items or numbers put in a pre-decided order

Term: a single number or variable

Position: the place something is located

Linear: the difference between terms increases or decreases (+ or -) by a constant value each time

Non-linear: the difference between terms increases or decreases in different amounts, or by x or \div

Difference: the gap between two terms

Linear and Non Linear Sequences

Linear Sequences — increase by addition or subtraction and the same amount each time

Non-linear Sequences — do not increase by a constant amount — quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence — look out for this type of sequence

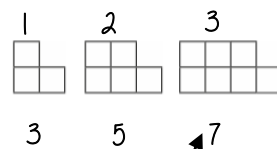
0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms



Sequence in a table and graphically

Position: the place in the sequence



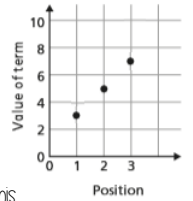
Term: the number or variable (the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** — as seen in the graph

"The term in position 3 has 7 squares"

Sequences from algebraic rules

This is substitution!

$$3n + 7$$

$$3n^2 + 7$$

This will be linear - note the single power of n . The values increase at a constant rate

This is not linear as there is a power for n

$$2n - 5 \longrightarrow$$

Substitute the number of the term you are looking for in place of 'n'

- eg
- 1st term = $2(1) - 5 = -3$
 - 2nd term = $2(2) - 5 = -1$
 - 100th term = $2(100) - 5 = 195$

Checking for a term in a sequence

Form an equation

Is 201 in the sequence $3n - 4$?

Algebraic rule

$$3n - 4 = 201$$

Term to check

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

Complex algebraic rules

Misconceptions and comparisons

$$2n^2$$

$$(2n)^2$$

2 times whatever n squared is

2 times n then square the answer

- eg
- 1st term = $2 \times 1^2 = 2$
 - 2nd term = $2 \times 2^2 = 8$
 - 100th term = $2 \times 100^2 = 2000$

- eg
- 1st term = $(2 \times 1)^2 = 4$
 - 2nd term = $(2 \times 2)^2 = 16$
 - 100th term = $(2 \times 100)^2 = 40000$

$$n(n + 5)$$

- eg
- 1st term = $1(1 + 5) = 6$
 - 2nd term = $2(2 + 5) = 14$
 - 100th term = $100(100 + 5) = 10500$

You don't need to expand the expression

H Finding the algebraic rule

This is the 4 times table \longrightarrow 4, 8, 12, 16, 20....

$$4n$$

7, 11, 15, 19, 22

This has the same constant difference — but is 3 more than the original sequence

$$4n + 3$$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

$$4n + 3$$

YEAR 8 — DEVELOPING NUMBER...

Block 10: Fractions & Percentages

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between FDP less than and more than 100.
- Increase or decrease using multipliers.
- Express an amount as a percentage.
- Find percentage change.

Keywords

Percent: parts per 100 — written using the % symbol

Decrease: to make smaller in value.

Increase: to increase/ to grow.

Integer: whole number, can be positive, negative or zero.

VAT: Value added tax, a percentage added onto a monetary amount

Convert FDP



70/100 → This also means 70 out of 100 squares → 70 hundredths = 70% → 0.7

Using a calculator → S-D → Convert to a decimal → × 100 converts to a percentage

This will give you the answer in the simplest form

Be careful of recurring decimals
 e.g. $\frac{1}{3} = 0.333333$
 $\frac{2}{3} = 0.\dot{3}$
 The dot above the 3

Fraction/ Percentage of amount



Find $\frac{3}{5}$ of £60 → £36

Remember $\frac{3}{5} = 60\%$

10% of £60 = £6
 50% of £60 = £30
 60% of £60 = £36

Remember $\frac{3}{5} = 60\% = 0.6$
 60% of £60 = 0.6 × 60 = £36

Convert FDP < and > 100%

100 hundredths = 100% → 140 hundredths = 140%

100% + 40% = 1 + 0.40 = 1.40 = 140%

Percentage decrease: Multipliers

100% → Decrease by 58% → 42%

100% - 58% = 42%

100 - 58 = 42

Multiplier Less than 1

Percentage increase: Multipliers

100% → Increase by 12% → 112%

100% + 12% = 112%

100 + 12 = 112

Multiplier More than 1

Express as a % - Non-calculator

7 per every 10 are orange → $\frac{7}{10}$ → This means that 70 per every 100 are orange → $\frac{70}{100}$ → 70%

27 per every 50 shaded → $\frac{27}{50}$ → 54 per every 100 shaded → $\frac{54}{100}$ → 54%

Denominator 100 Equivalent fractions

Express as a % - Calculator

Rosie $\frac{13}{30}$ → $\frac{13}{30}$ → × 100 → 43.333...% → 43%

Can't use equivalence easily to find 'per hundred'

This is the same as 13 ÷ 30

Decimal percentages are still a percentage.

Percentage change

I bought a phone for £200. A year later sold it for £125.

Percentage loss: $\frac{75}{200} \times 100 = 37.5\%$

All values of change compare to the ORIGINAL value.

I bought a house for £180,000, I later sold it for £216,000.

Percentage profit: $\frac{36000}{180000} \times 100 = 20\%$

Money made (profit value)

Choose appropriate method

The language and wording of the question is the key.

Have you represented the question in a bar model?
 Can you use a calculator?

YEAR 8 — DEVELOPING NUMBER...

Block 1: Number Sense and standard form

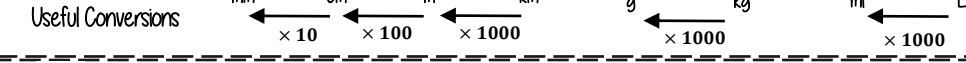
What do I need to be able to do?

- By the end of this unit you should be able to:
- Round numbers to powers of 10 and 1 sf
 - Round numbers to any dp
 - Estimate solutions
 - Calculate using order of operations
 - Calculate with money, units of measurement and time
 - Write numbers in standard form and as ordinary numbers

Keywords

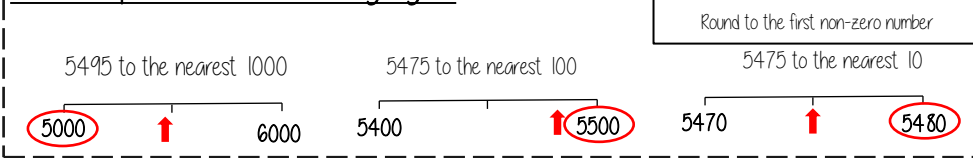
- Significant:** Place value of importance
- Base:** The number that gets multiplied by a power
- Power:** The exponent — or the number that tells you how many times to use the number in multiplication
- Indices:** The power or the exponent
- Estimate:** A good mathematical guess when numbers are rounded sensibly

Units are important:



Round to powers of 10 and 1 sig. figure

R If the number is halfway between we "round up"



- 370 to 1 significant figure is 400
- 37 to 1 significant figure is 40
- 3.7 to 1 significant figure is 4
- 0.37 to 1 significant figure is 0.4
- 0.00037 to 1 significant figure is 0.0004

Round to decimal places

"To 1dp" — to one number after the decimal
 "To 2dp" — to two numbers after the decimal

2.46192 (to 1dp) — Is this closer to 2.4 or 2.5
 2.46192 (to 2dp) — Is this closer to 2.46 or 2.47

Estimate the calculation

Round to 1 significant figure to estimate

$4.2 + 6.7 \approx 4 + 7 \approx 11$ This is an **overestimate** because the 6.7 was rounded up more

$214 \times 3.1 \approx 20 \times 3 \approx 60$ This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths — it helps you identify calculation errors

Order of operations

- Brackets** Operations in brackets are calculated first
- Other** operations e.g. powers, roots,
- Multiplication/ Division** They are carried out in the order from left to right in the question
- Addition/ Subtraction** They are carried out in the order from left to right in the question

Calculations with money

Debit — You have £0 or more in an account
Credit — You have less than £0 in an account

Using a calculator — ensure you are working in the correct units

$£130 + 50p = 130 + 50$ (in pence)
 $= 130 + 0.50$ (in pounds)

Money calculations are to 2dp

Time and the calendar

- 1 Year** — the amount of time it takes Earth to go around the sun **365** (and a quarter) days
- 1 week** — 7 days
Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday
- 1 day** — 24 hours
- 1 hour** — 60 minutes
- 1 minute** — 60 seconds
- Leap Year** — 366 days (every 4 years)
- 12 Months** — one year = 52 weeks
- 31 days — Jan, March, May, July, Aug, Oct, Dec
- 30 days — April, June, Sept, Nov
- 28 days — Feb (29 leap year)

Use a number line for time calculations!

Positive powers of 10

1 billion — 1 000 000 000
 $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

Any number between 1 and less than 10 $\rightarrow A \times 10^n$ (Any integer)

Example
 3.2×10^4
 $= 3.2 \times 10 \times 10 \times 10 \times 10$
 $= 32000$

Non-example
 0.8×10^4
 $5.3 \times 10^{0.7}$

Negative powers of 10

0.001	10^2	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
$1 \times \frac{1}{1000}$	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1×10^{-3}	0	0	0	0	0

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054
 $= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}			
0.064	240	1	0.13			

Look at the power first will the number be $>$ or $<$ than 1

Use a place value grid to compare the numbers for ordering

A negative power does not mean a negative answer — it means a number closer to 0

YEAR 8 — DEVELOPING GEOMETRY...

Block 12: Angles in parallel lines and polygons

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify alternate angles
- Identify corresponding angles
- Identify co-interior angles
- Find the sum of interior angles in polygons
- Find the sum of exterior angles in polygons
- Find interior angles in regular polygons

Keywords

Parallel: Straight lines that never meet

Transversal: A line that cuts across two or more other (normally parallel) lines

Isosceles: Two equal size lines and equal size angles (in a triangle or trapezium)

Polygon: A 2D shape made with straight lines

Vertex: a point where two or more-line segments meet

Basic angle rules and notation R

Acute Angles
 $0^\circ < \text{angle} < 90^\circ$

Right Angles
 90°

Obtuse
 $90^\circ < \text{angle} < 180^\circ$

Reflex
 $180^\circ < \text{angle} < 360^\circ$

Straight Line
 180°

Right angle notation

The letter in the middle is the angle
 The arc represents the part of the angle

Angle Notation: three letters ABC
 This is the angle at B = 113°

Line Notation: two letters EC
 The line that joins E to C

Vertically opposite angles
 Equal
Angles around a point
 360°

Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!

Lines OF and BE are transversals (lines that bisect the parallel lines)

Corresponding angles often identified by their "F shape" in position

Alternate angles often identified by their "Z shape" in position

This notation identifies parallel lines

Alternate/ Corresponding angles

Because alternate angles are equal the highlighted angles are the same size

Because corresponding angles are equal the highlighted angles are the same size

Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°

Os angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Triangles & Quadrilaterals R

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side

Link to steps R

Properties of Quadrilaterals

Square
 All sides equal size
 All angles 90°
 Opposite sides are parallel

Rectangle
 All angles 90°
 Opposite sides are parallel

Rhombus
 All sides equal size
 Opposite angles are equal

Parallelogram
 Opposite sides are parallel
 Opposite angles are equal
 Co-interior angles

Trapezium
 One pair of parallel lines

Kite
 No parallel lines
 Equal lengths on top sides
 Equal lengths on bottom sides
 One pair of equal angles

Sum of exterior angles

Exterior angles all add up to 360°

Using exterior angles

Interior angle + Exterior angle = straight line = 180°
 Exterior angle = $180 - 165 = 15^\circ$

Number of sides = $360^\circ \div \text{exterior angle}$
 Number of sides = $360 \div 15 = 24$ sides

Sum of interior angles

Interior Angles
 The angles enclosed by the polygon

$(\text{number of sides} - 2) \times 180$

Sum of the interior angles = $(5 - 2) \times 180$

This shape can be made from three triangles
 Each triangle has 180°

Sum of the interior angles = $3 \times 180 = 540^\circ$

Remember this is all of the interior angles added together

Missing angles in regular polygons

Exterior angle = $360 \div 8 = 45^\circ$

Interior angle = $\frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8} = 135^\circ$

Exterior angles in regular polygons = $360^\circ \div \text{number of sides}$

Interior angles in regular polygons = $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

YEAR 8 — DEVELOPING GEOMETRY...

Block 13: Area of trapezia and Circles

What do I need to be able to do?

By the end of this unit you should be able to:

- Recall area of basic 2D shapes
- Find the area of a trapezium
- Find the area of a circle
- Find the area of compound shapes
- Find the perimeter of compound shapes

Keywords

Congruent: The same

Area: Space inside a 2D object

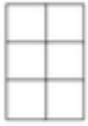
Perimeter: Length around the outside of a 2D object

Perpendicular: At an angle of 90° to a given surface

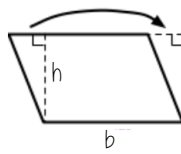
Formula: A mathematical relationship/ rule given in symbols. E.g $b \times h = \text{area of rectangle/ square}$

Area — rectangles, triangles, parallelograms

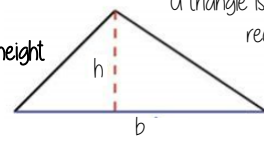
Rectangle
Base x Height



Parallelogram/ Rhombus
Base x Perpendicular height



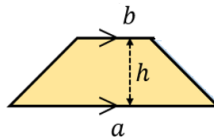
Triangle
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$



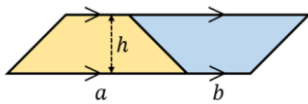
A triangle is half the size of the rectangle it would fit in

Area of a trapezium

Area of a trapezium
 $\frac{(a+b) \times h}{2}$



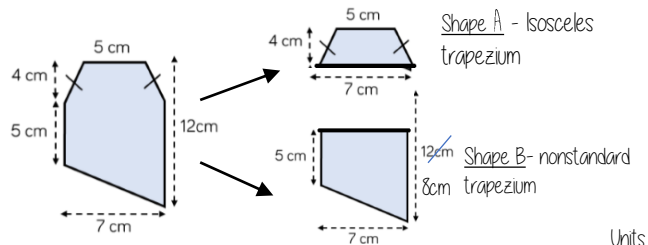
Why?



- Two congruent trapeziums make a parallelogram
- New length $(a + b) \times \text{height}$
- Divide by 2 to find area of one

Compound shapes

To find the area compound shapes often need splitting into more manageable shapes first. Identify the shapes and missing sides etc. first.



Shape A + Shape B = total area

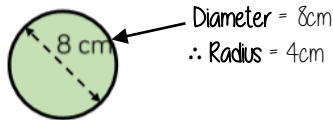
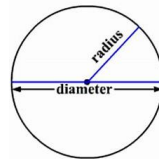
$$\frac{(5+7) \times 4}{2} + \frac{(5+7) \times 8}{2} = 24 + 45.5 = 69.5 \text{ cm}^2$$

Units

Area of a circle (Non-Calculator)

Read the question — leave in terms of π or if $\pi \approx 3$ (provides an estimate for answers)

Area of a circle
 $\pi \times \text{radius}^2$



Diameter = 8cm
 \therefore Radius = 4cm

$$\begin{aligned} \pi \times \text{radius}^2 \\ = \pi \times 4^2 \\ = \pi \times 16 \\ = 16\pi \text{ cm}^2 \end{aligned}$$

Find the area of one quarter of the circle



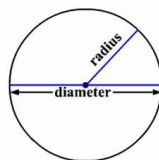
Circle Area = $16\pi \text{ cm}^2$
Quarter = $4\pi \text{ cm}^2$

Area of a circle (Calculator)



SHIFT $\times 10^x$

Area of a circle
 $\pi \times \text{radius}^2$



How to get π symbol on the calculator

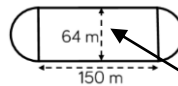
It is important to round your answer suitably — to significant figures or decimal places. This will give you a decimal solution that will go on forever!

Compound shapes including circles

Circumference
 $\pi \times \text{diameter}$

Compound shapes are not always area questions
For Perimeter you will need to use the circumference

Spotting diameters and radii



This dimension is also the diameter of the semi circles

$$\begin{aligned} \text{Arc lengths} &= \pi \times 64 \\ &= 64\pi \end{aligned}$$

Don't need to halve this because there are 2 ends which make the whole circle

Arc lengths + Straight lengths = total perimeter

$$\begin{aligned} &= 64\pi + 150 + 150 \\ &= (300 + 64\pi) \text{ m} \\ \text{OR} &= 501.1 \text{ m} \end{aligned}$$

Still remember to split up the compound shape into smaller more manageable individual shapes first

YEAR 8 — DEVELOPING GEOMETRY...

Block 14: Lines of symmetry and reflection

What do I need to be able to do?

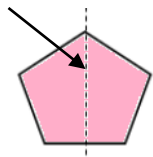
- By the end of this unit you should be able to:
- Recognise line symmetry
 - Reflect in a horizontal line
 - Reflect in a vertical line
 - Reflect in a diagonal line

Keywords

- Mirror line:** a line that passes through the center of a shape with a mirror image on either side of the line.
Horizontal: a straight line from left to right (parallel to the x axis)
Vertical: a straight line from top to bottom (parallel to the y axis)
Object: The original/starting shape
Image: The shape after a reflection has taken place

Lines of symmetry

Mirror line (line of reflection)



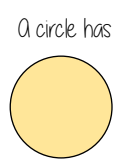
Shapes can have more than one line of symmetry...
 This regular polygon (a regular pentagon has 5 lines of symmetry)



Rhombus
two lines of symmetry

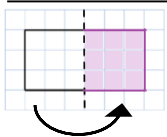
Parallelogram

No lines of symmetry



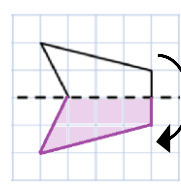
A circle has an infinite amount of lines of symmetry

Reflect horizontally/ vertically (1)



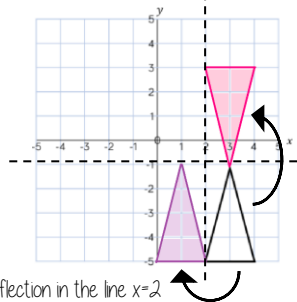
Reflection in a vertical line

Note a reflection doubles the area of the original shape



Reflection in a horizontal line

Reflection on an axis grid

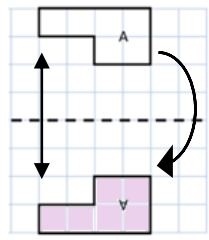


Reflection in the line $y = -2$

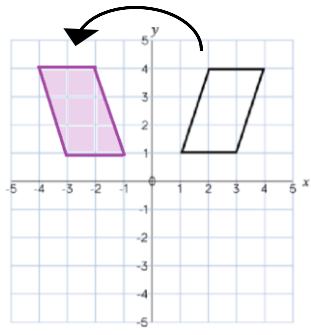
Reflection in the line $x = 2$

Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



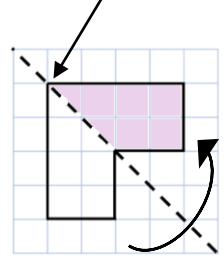
Reflection in the line y axis — this is also a reflection in the line $x = 0$



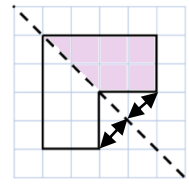
Lines parallel to the x and y axis
 REMEMBER
 Lines parallel to the x-axis are $y = \dots$
 Lines parallel to the y-axis are $x = \dots$

Reflect Diagonally (1)

Points on the mirror line don't change position

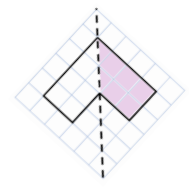


Fold along the line of symmetry to check the direction of the reflection



Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

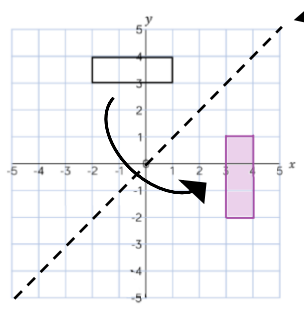


Drawing perpendicular lines

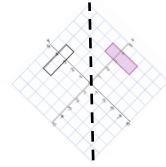
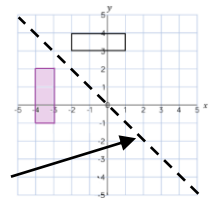
Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

Reflect Diagonally (2)

This is the line $y = x$ (every y coordinate is the same as the x coordinate along this line)



This is the line $y = -x$
 The x and y coordinate have the same value but opposite sign



Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

YEAR 8 — REASONING WITH DATA...

Block 15: The data handling cycle

What do I need to be able to do?

By the end of this unit you should be able to:

- Set up a statistical enquiry
- Design and criticise questionnaires
- Draw and interpret multiple bar charts
- Draw and interpret line graphs
- Represent and interpret grouped quantitative data
- Find and interpret the range
- Compare distributions

Keywords

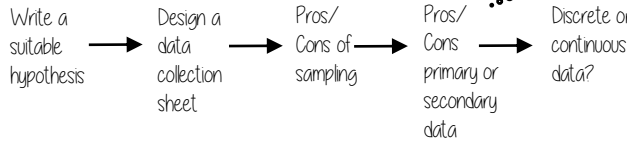
Hypothesis: an idea or question you want to test

Sample: the group of things you want to use to check your hypothesis

Primary Data: data you collect yourself

Secondary Data: data you source from elsewhere e.g the internet/ newspapers/ local statistics

Set up a statistical enquiry



Features of a data collection sheet

Data Title	Tally	Frequency

Total number of that group observed

Design and criticise a questionnaire

The Question - be clear with the question - don't be too leading/ judgemental

e.g How much pocket money do you get a week?

Responses - do you want closed or open responses? - do any options overlap? - Have you an option for all responses?

Zero option → £0 £0.01 - £2 £2.01 - £4 more than £4 ← More option

NOTE: For responses about continuous data include inequalities $< x \leq$

Pictograms, bar and line charts

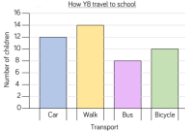
Pictogram

Language	
French	●●●●●
Spanish	●●●●●
German	●●●●●

● = 4 people

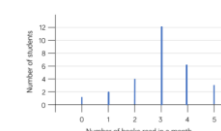
- Need to remember a key
- Visually able to identify mode

Bar Chart



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

Line Chart



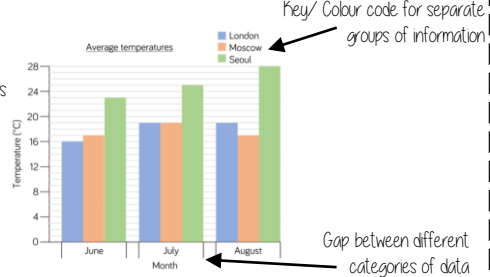
- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

Represents quantitative data

Multiple Bar chart

Compares multiple groups of data

- Clearly labelled axes
- Scale for axes
- Comparable data bars drawn next to each other



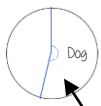
Draw and interpret Pie Charts

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

Multiple method

As 60 goes into 360 - 6 times
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)



Use a protractor to draw
This is 192°

Represents quantitative, discrete data

This fraction of the 360 degrees represents dogs

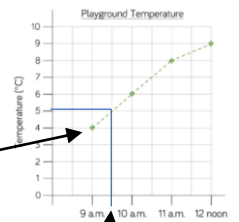
$$\frac{32}{60} \times 360 = 192^\circ$$

Draw and interpret line graphs

- Commonly used to show changing over time
- The points are the recorded information and the lines join the points

Line graphs do not need to start from 0

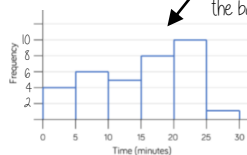
More than one piece of data can be plotted on the same graph to compare data



It is possible to make estimates from the line
e.g temperature at 9.30am is 5°C

Grouped quantitative data

Time (minutes)	Frequency
$0 \leq t < 5$	4
$5 \leq t < 10$	6
$10 \leq t < 15$	5
$15 \leq t < 20$	8
$20 \leq t < 25$	10
$25 \leq t < 30$	1



This is a frequency diagram
There are no gaps between the bars

Grouping the data is useful if there is a large spread of data to begin with

The use of inequalities shows that this will be a frequency diagram

"More than or equal to 25 and less than 30 minutes"

Find and interpret the range

The range is a measure of spread

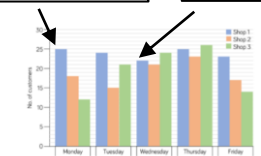
A smaller range means there is less variation in the results - it is more consistent data

A range of 0 means all the data is the same value

Shop 1 has the smallest range - this indicates it has a more consistent flow of customers each week.

Difference between the biggest and smallest values

Shop 1 highest value Shop 1 lowest value



Range of customers = $25 - 22 = 3$ (Shop 1)

YEAR 8 — REASONING WITH DATA...

Block 16: Measures of location

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and use mean, median and mode
- Choose the most appropriate average
- Identify outliers
- Compare distributions using averages and range

Keywords

Spread: the distance/ how spread out/ variation of data

Total: all the data added together

Frequency: the number of times the data values occur

Outlier: a value that stands apart from the data set

Mean, Median, Mode

The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values) 55

Divide the overall total by how many pieces of data you have $55 \div 5$

Mean = 11

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order

4, 8, 8, 11, 24

Find the value in the middle

4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value, find the mean of the two numbers left

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

This can still be easier if the data is ordered first

4, 8, 8, 11, 24

Mode = 8

Choosing the appropriate average

The average should be a representative of the data set — so it should be compared to the set as a whole - to check if it is an appropriate average

Here are the weekly wages of a small firm

£240 £240 £240 £240 £240
£260 £260 £300 £350 £700

Which average best represents the weekly wage?

The Mean = £307

The Median = £250

The Mode = £240

Put the data back into context

Mean/Median — too high (most of this company earn £240)

Mode is the best average that represents this wage

It is likely that the salaries above £240 are more senior staff members — their salary doesn't represent the average weekly wage of the majority of employees

Identify outliers

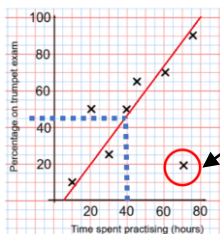
Outliers are values that stand well apart from the rest of the data

Outliers can have a big impact on range and mean. They have less impact on the median and the mode

Sometimes it is best to not use an outlier in calculations

Height in cm
152 150 142 158 182 151 153 149 156 160 151 144

Where an outlier is identified try to give it some context. This is likely to be a taller member of the group. Could it be an older student or a teacher?



Outliers can also be identified graphically e.g. on scatter graphs

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency

Here are the number of runs scored last month by Lucy and James in cricket matches

Lucy: 45, 32, 37, 41, 48, 35
James: 60, 90, 41, 23, 14, 23

Lucy

Mean: 39.6 (1dp), Median: 38, Mode: no mode, Range: 16

James

Mean: 41.8 (1dp), Median: 32, Mode: 23, Range: 76

James has two extreme values that have a big impact on the range

"James is less consistent than Lucy because his scores have a greater range. Lucy performed better on average because her scores have a similar mean and a higher median"